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Modulational instability of electron-acoustic waves: an application to auroral zone plasma

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Abstract. By using the standard reductive perturbation technique, a nonlinear Schrödinger equation is derived to study the modulational instability of finite amplitude electron-acoustic waves in an unmagnetized plasma consisting of cold electron fluid and nonthermal electrons. It is found that the presence of nonthermally distributed electrons modifies the domain of the modulational instability and solitary structures. Possibility of stationary states of the wave packets that can appear as envelope solitons under different conditions is explored. The present investigation is relevant to observation from the Viking satellite in the dayside auroral zone.

PACS. 52.35.-g Waves, oscillations, and instabilities in plasmas and intense beams

1 Introduction

Nonlinear behaviour of natural phenomena is one of the most important subject of plasma physics. In the nonlinear wave studies, the propagation of solitary waves is important as it describes the characteristics of interaction between waves and plasma. Among the best known paradigms used to investigate nonlinear wave behaviour are different versions of Korteweg-de Vries (KdV) equation, or nonlinear Schrödinger equation (NLSE). Some form of reductive perturbation technique is used to derive such equations. The KdV equation describes the evolution of unmodulated wave. In this case, the bare pulse does not contain high frequency oscillations inside the packet, is called KdV soliton. On the other hand, the NLSE governs the dynamics of a modulated wave packet. Here the nonlinearities are in balance with wave group dispersion and the resulting solution of the NLSE possesses envelope structure, known as an envelope soliton.

Electron-acoustic (EA) wave is an electrostatic wave which had been first discovered experimentally [1–3]. The plasma with two population groups of electrons, described by two Maxwellian distribution functions with different temperature, supports an electrostatic electron-acoustic wave [4]. The two populations are referred to as 'cold' and 'hot' electrons with respective temperatures T_c and T_h. The cold electrons in EA waves play the role of cold ions in ion-acoustic waves and in this sense the wave is analogous to ion-acoustic wave. The ions play no role in the dynamics of EA waves and are required solely for charge neutralization. The mode is restrictive in the sense that it demands $T_c \ll T_h$ and further the hot electron population should represent a significant fraction of total electron density. It may further be noted that plasmas with different temperatures and masses frequently occur in the space environment, particularly, the two temperature electrons are very common in laboratory [5] and space plasmas. Interestingly, the studies of two electron temperatures are encouraged by satellite observations [6–10]. The nonlinear localized structures, e.g., ion-acoustic solitons and double-layers in two electrons component plasma, have been studied experimentally and theoretically by a number of authors. Since plasmas with two electrons temperatures occur in both laboratory experimental and space plasmas, Gary and Tokar [11] performed a parameter survey and found conditions for the existence of the EA waves. The EA mode plays an important role in these environments [12]. In the earth's bow shock, particularly in the upstream region, the electron acoustic waves have been suggested as a possible source of broadband electrostatic noise (BEN). They are also of potential importance in interpreting BEN observed in cusp of terrestrial magnetospace in auroral region and in geomagnetic tail [12–15]. The EA mode has also been used to explain wave emissions in different regions of the Earth's magnetosphere. Furthermore, the EA mode has been applied to interpret the hiss observed in the polar cusp region.

A study of nonlinear properties of large amplitude necessarily useful for understanding BEN, was pointed by Mace et al. [16]. Dubouloz et al. [17,18] rigorously studied the BEN observed in the dayside of auroral zone and explained short duration burst of BEN in terms of EA

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solitary waves. They considered a one-dimensional unmagnetized collisionless plasma consisting of cold electrons, Maxwellian hot electrons and stationary ions. In the presence of large electron beam energy, the nonlinear effects combine with the dispersive properties of EA waves resulting in the formation of EA solitons [12]. This leads to existence of new EA solitons with velocity related to beam velocity. Berthomier et al. [12] pointed out that positive potential structure is very important from the point of view of the interpretation of various electrostatic structures observed in the auroral region at intermediate altitude by FAST and at higher altitudes by POLAR and in geomagnetic tail by GEOTAIL. Mace and Hellberg [19] studied the effect of magnetic field on electron acoustic solitons. They derived KdV-ZK equation for weakly nonlinear EA waves and discussed its solitonic solutions.

In practice, the hot electrons may not follow a Maxwellian distribution. Moreover, it had been found that the electrons and ions distributions play a crucial role in characterizing the physics of nonlinear waves. They offer a considerable increase in richness and variety of wave motion which can exist in plasma and further significantly influence the conditions required for the formation of these waves. Moreover, it is also known that electron and ion distributions can be significantly modified in the presence of large amplitude waves. Cairns et al. [20,21] explained the structure of solitary waves with density depression using nonthermal distribution for electrons. The properties of small but finite EA solitary waves were studied in a plasma with cold electron fluid, hot electrons obeying a trapped/vortex-like distributions and stationary ion [22]. Nonthermal distributions are common feature of the auroral zone [23,24]. It may be mentioned here that the origin and mechanism for the generation of nonthermal particles in space plasma is still a central problem.

For envelope soliton, there has been an increased interest in recent years on the investigation of modulational instability of different wave modes in plasma because of its importance in stable wave propagation. However, only a few investigations are reported for ion-acoustic mode [25–31]. It is further observed that EA waves being high frequency density waves, are trapped and modulated leading to modulation and generation of electron-acoustic envelope solitons. In high time resolution of the FAST observations, these kinds of nonlinear structures are observed [32]. Most of the investigations reported so far, have been restricted to modulational instability of ion-acoustic waves in plasma with two temperature electrons [27]. In the present investigation, we study the modulational instability of EA waves in plasma with nonthermal electrons. We have used the range of parameters of auroral zone plasma measured from Viking satellite [10,17]. Using the reductive perturbation technique, we have derived the NLSE, which governs the slow modulation of the wave amplitude. In Section 2, we have introduced basic equations governing the dynamics of EA mode and derived the nonlinear schrödinger equation using reductive perturbation method. Stability analysis and discussion of the results are presented in the last section.

2 Derivation of nonlinear Schrödinger equation using reductive perturbation technique

Since the plasmas with two electrons population are known to occur frequently in space, EA wave may play an important role in such environment. A collisionless infinite homogeneous and unmagnetized plasma is considered in a following model. The plasma fluid model consists of cold and hot components referred to subscript c and h respectively. The presence of two nondrifting populations allows the existence of the EA waves itself [12]. It may be mentioned that cold electron component does not mean $T_c = 0$. In that case EA wave will not exist [12]. The wave propagation is assumed to be in one direction, which we choose to be along x-axis. The fluid equations of cold electron component and Poisson's equation can be written as follow:

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u_c}{\partial t} + u_c\frac{\partial u_c}{\partial x} + \frac{3\alpha(1+\alpha)^2 n_c}{\theta}\frac{\partial n_c}{\partial x} - \alpha\frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha}n_c + n_h - \left(1 + \frac{1}{\alpha}\right) \tag{3}$$

where n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium value $n_{co}(n_{ho})$. Here ϕ is the electrostatic wave potential normalized by $k_B T_h/e$, u_c is the electron fluid velocity normalized by $(k_B T_h/\alpha m)^{1/2}$, $\alpha = n_{ho}/n_{co}$, m is the mass of the electron, $\theta = T_h/T_c$, e is the electron charge and k_B is Boltzmann constant, T_h is the temperature of hot electrons. The space and time variables are in units of hot electron Debye length $(K_B T_h/4\pi n_{ho} e^2)^{1/2}$ and cold electron plasma period $\omega_{pc}^{-1} = (m/4\pi n_{co} e^2)^{1/2}$ respectively.

In (2), inertia of cold electron is included and cold population is assumed to respond adiabatically to electric field perturbation. The physical origin of third term in (2) is pressure term where adiabatic cold electrons are considered and ratio of specific heats taken as three.

It may be mentioned that $T_c \neq 0$ [33,34]. As earlier mentioned in the introduction , the conditions for the existence of EA mode are as follow:

(i) $T_h \gg T_c$;
(ii) cold electrons represent a significant fraction of plasma (more than 20%). Thus, θ can not be zero for the existence of EA mode.

Following the model of Cairns et al. [20,21], the nonthermal distribution for the electron is taken as

$$f(v) = \frac{1}{(1+3\gamma)\sqrt{2\pi}}(1+\gamma v^4)e^{-v^2/2}. \tag{4}$$

The real parameter γ is an arbitrary parameter which defines the shape of the distribution function and expresses the deviation from the Maxwellian state. This form of the distribution is convenient for the description of various observed particle distributions. For example, when $\gamma = 0$, we

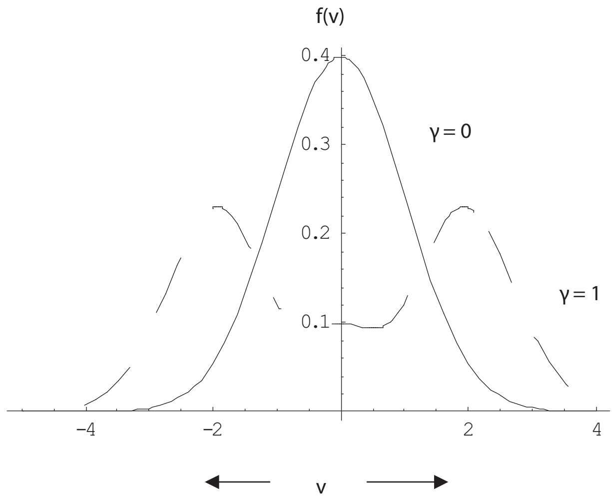

Fig. 1. Variation of distribution function $f(v)$ as a function of v for two values of γ.

get Maxwellian distribution and when $\gamma \to 1$, it tends to look as two counter streaming beams with cold core distribution [35]. This last aspect is apparent from Figure 1, which highlights this feature of the distribution. Observation is contrary to laser cooling where distribution is compressed in such a way that very large numbers of atoms contain low velocities. In the present case large number of low velocity particles are symmetrically displaced to form counter streaming beams. As earlier mentioned, the mechanism for this process is not clear. For the effects of electrostatic disturbance on the electron distribution, we replace v^2 by $(v^2 - 2\phi)$ and on performing the integration $n_h = \int_{-\infty}^{\infty} f(v)dv$, we get

$$n_h = (1 - \beta\phi + \beta\phi^2)e^{\phi} \tag{5}$$

where $\beta = 4\gamma/(1 + 3\gamma)$. The parameter β represents the nonthermality of hot electron distribution.

Modulational instability of electron-acoustic wave is studied using reductive perturbation technique. The aim of the present work is to derive nonlinear Schrödinger equation which governs the slow modulation of wave amplitude. The method is based on reductive perturbation technique in which dependent variables are expanded and which also make use stretching of space and time coordinates. We have made use of the following set of stretched (slow) space and time variables

$$\xi = \epsilon(x - v_g t) \tag{6}$$

$$\tau = \epsilon^2 t \tag{7}$$

where v_g is the group velocity to be determined by the compatability requirement. Here ϵ is a small formal expansion parameter and is the measure of the perturbation. The condition $\epsilon \ll 1$ implies that the plasma dimension must be much larger than the Debye length, which is satisfied in the most cases of interest. We will assume that all perturbed quantities depend on the fast scale via the phase $\chi = kx - \omega t$ only, while the slow scales enter the argument of the lth harmonic amplitude, say for density as $n_l^{(n)}$. Following this prescription, the dependent variables are expanded as

$$n_c = 1 + \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \epsilon^n n_l^n(\xi, \tau) e^{\iota l(kx - \omega t)} \tag{8}$$

$$u_c = \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \epsilon^n u_l^n(\xi, \tau) e^{\iota l(kx - \omega t)} \tag{9}$$

$$\phi = \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \epsilon^n \phi_l^n(\xi, \tau) e^{\iota l(kx - \omega t)} \tag{10}$$

where n_c, u_c, ϕ satisfy the reality condition $A_{-l}^{(n)} = A_l^{(n)*}$ and asterisk denote the complex conjugate. Using (5) through (10) in (1) to (3) and collecting the terms of different powers of ϵ, we get the reduced equations. For the first order ($n = 1$), we get

$$-\iota\omega n_1^{(1)} + \iota k u_1^{(1)} = 0 \tag{11}$$

$$-\iota\omega u_1^{(1)} + \frac{3\iota k\alpha}{\theta}(1 + \alpha)n_1^{(1)} - \iota k\alpha\phi_1^{(1)} = 0 \tag{12}$$

$$(1 - \beta + k^2)\phi_1^{(1)} + \frac{1}{\alpha}n_1^{(1)} = 0. \tag{13}$$

Algebraic manipulations of these equations lead to the following dispersion relation

$$\omega^2 = \frac{k^2}{1 - \beta + k^2} + \frac{3k^2\alpha(1 + \alpha)}{\theta}. \tag{14}$$

From (11) to (13), we can express the first order quantities in terms of $\phi_1^{(1)}$ as

$$n_1^{(1)} = -\alpha(1 - \beta + k^2)\phi_1^{(1)} \tag{15}$$

$$u_1^{(1)} = -\alpha\frac{\omega}{k}(1 - \beta + k^2)\phi_1^{(1)}. \tag{16}$$

For the second order ($n = 2$), reduced equation with $l = 1$, we get

$$-\iota\omega n_1^{(2)} + \iota k u_1^{(2)} = v_g\frac{\partial n_1^{(1)}}{\partial\xi} - \frac{\partial u_1^{(1)}}{\partial\xi} \tag{17}$$

$$-\iota\omega u_1^{(2)} + \frac{3\iota k\alpha}{\theta}(1 + \alpha)n_1^{(2)} - \iota k\alpha\phi_1^{(2)} = v_g\frac{\partial u_1^{(1)}}{\partial\xi} - \frac{3\alpha(1 + \alpha)}{\theta}\frac{\partial n_1^{(1)}}{\partial\xi} + \alpha\frac{\partial\phi_1^{(1)}}{\partial\xi} \tag{18}$$

$$(1 - \beta + k^2)\phi_1^{(2)} + \frac{1}{\alpha}n_1^{(2)} = 2\iota k\frac{\partial\phi_1^{(1)}}{\partial\xi}. \tag{19}$$

Using (15) to (19), we can put the second order quantities $n_1^{(2)}$, $u_1^{(2)}$ in terms of $\phi_1^{(2)}$ and $\partial\phi_1^{(1)}/\partial\xi$. Then, these are further algebraically manipulated and we obtain the following compatability condition:

$$v_g = \frac{k}{\omega}\left[\frac{1 - \beta}{(1 - \beta + k^2)^2} + \frac{3\alpha(1 + \alpha)}{\theta}\right]. \tag{20}$$

$$P = -\frac{3}{2}\frac{k^4}{\omega^3(1-\beta+k^2)^4}\left[(1-\beta)+\left(\frac{3\alpha(1+\alpha)(1-\beta)}{\theta}-\frac{k^2\alpha(1+\alpha)}{\theta}\right)(1-\beta+k^2)\right] \tag{30}$$

$$\begin{aligned}Q = \frac{k}{2\alpha\omega^2(1-\beta+k^2)}&\left[-Bk\omega\left(\alpha+\frac{1}{(1-\beta)(1-\beta+k^2)}+\frac{2\alpha(1-\beta)}{1-\beta+k^2}+\frac{12\alpha^2(1+\alpha)(1-\beta+k^2)}{\theta}\right)\right.\\ &-k^2A\left[\frac{1}{(1-\beta+k^2)(1-\beta+4k^2)}+3\alpha\left(1+\frac{4\alpha(1+\alpha)(1-\beta+k^2)}{\theta}\right)\right]\\ &+3\omega k(1-\beta+k^2)^2\left(1+\frac{2\alpha(1+\alpha)(1-\beta+k^2)}{\theta}\right)\\ &\left.+\frac{\omega k\alpha(1+3\beta)}{2(1-\beta+k^2)}-\frac{k\alpha\omega}{(1-\beta)(1-\beta+k^2)}-\frac{k\omega(2\alpha(1-\beta+k^2)^2-1)}{2(1-\beta+k^2)(1-\beta+4k^2)}\right]\end{aligned} \tag{31}$$

The second harmonic mode of the carrier, which comes from nonlinear self-interaction, is also obtained in terms of $[\phi_1^{(1)}]^2$. The component $l=2$ for the second order, $n=2$, reduced equations determine the second order quantities. They turn out to be

$$u_2^{(2)} = D[\phi_1^{(1)}]^2 \tag{21}$$

$$n_2^{(2)} = \left[\frac{k}{\omega}D+(1-\beta+k^2)^2\alpha^2\right][\phi_1^{(1)}]^2 \tag{22}$$

$$\phi_2^{(2)} = \frac{\left[-\frac{2}{\alpha}\left(\frac{k}{\omega}D+(1-\beta+k^2)^2\alpha^2\right)-1\right]}{2(1-\beta+4k^2)}[\phi_1^{(1)}]^2 \tag{23}$$

where

$$\begin{aligned}D = \frac{\alpha\omega(1-\beta+k^2)}{6k^3}&\left[\alpha(1-\beta+k^2)(1-\beta+4k^2)\right.\\ &+\frac{12\alpha^2}{\theta}(1+\alpha)(1-\beta+k^2)(1-\beta+4k^2)\\ &\left.+2\alpha(1-\beta+k^2)^2+1\right].\end{aligned} \tag{24}$$

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth order harmonic. Its strength is analytically determined by taking $l=0$ component of the third - order reduced equations i.e., for $n=2$, $l=0$. The result is expressed in terms of the square of modulus of $n=1$, $l=1$ quantities i.e., $[\phi_1^{(1)}]^2 = \phi_1^{(1)}\phi_1^{(1)*}$

$$n_0^{(2)} = B[\phi_1^{(1)}]^2 \tag{25}$$

$$\phi_0^{(2)} = -\left(\frac{1}{1-\beta}\right)\left\{\frac{B}{\alpha}+1\right\}^2[\phi_1^{(1)}]^2 \tag{26}$$

$$u_0^{(2)} = \left(Bv_g-\frac{2\omega}{k}(1-\beta+k^2)^2\alpha^2\right)[\phi_1^{(1)}]^2 \tag{27}$$

where

$$B = \frac{-\alpha\left[\frac{2\alpha}{k}(1-\beta)(1-\beta+k^2)^2\left(\omega v_g+\frac{3\alpha(1+\alpha)k}{\theta}\right)+\alpha(1-\beta+k^2)(1-\beta)+1\right]}{\left[1-(1-\beta)\left(v_g^2-\frac{3\alpha(1+\alpha)}{\theta}\right)\right]}. \tag{28}$$

Finally, substituting the above derived expressions into $l=1$ component of the third order ($n=3$) part of the reduced equation, we obtain the following nonlinear Schrödinger equation (NLSE):

$$\iota\frac{\partial\phi}{\partial\tau}+P\frac{\partial^2\phi}{\partial\xi^2}+Q|\phi|^2\phi = 0 \tag{29}$$

where

see equations (30, 31) above

with

$$\begin{aligned}A = \frac{\alpha\omega(1-\beta+k^2)}{6k^3}&\left[\alpha(1-\beta+k^2)(1-\beta+4k^2)\right.\\ &+\frac{12\alpha^2}{\theta}(1+\alpha)(1-\beta+k^2)(1-\beta+4k^2)\\ &\left.+2\alpha(1-\beta+k^2)^2+1\right].\end{aligned} \tag{32}$$

In the NLSE (29), we have replaced $\phi_1^{(1)}$by ϕ for the sake of notational convenience.

3 Stability analysis and discussion

In the standard stability analysis, we linearize around the monochromatic wave solution of the NLSE and modulation on the wave amplitude takes place in the propagation direction. Therefore, we separate the amplitude ϕ into two parts as follows:

$$\phi = [\Phi_0+\delta\phi(\zeta)]e^{(-\iota\Delta\tau)} \tag{33}$$

where $\zeta = K\xi-\Omega\tau$ is the modulation phase and $0<K\ll k$ and $\Omega\ll\omega$ are respectively the wavenumber and frequency of modulation. Φ_0 is the amplitude of pump

carrier wave, $\delta\phi \ll \Phi_0$ small amplitude perturbation and Δ is a nonlinear frequency shift.

Substituting (33) into (29) and collecting the terms of same order, we obtain

$$\Delta = -Q|\Phi_0|^2 \tag{34}$$

and

$$\iota\frac{\partial\delta\phi}{\partial\tau} + P\frac{\partial^2\delta\phi}{\partial\zeta^2} + Q|\Phi_0|^2(\delta\phi + \delta\phi^*) = 0 \tag{35}$$

where $\delta\phi^*$ is complex conjugate of $\delta\phi$. On assuming that the amplitude perturbation varies as $\exp[\iota(K\xi - \Omega\tau)]$ and following the standard procedure [36], after simplification we get

$$\Omega^2 = PK^2(PK^2 - 2Q|\Phi_0|^2). \tag{36}$$

Equation (36) is nonlinear dispersion relation for the amplitude modulation. It is apparent from this relation that $\Omega^2 > 0$ for all $k > 0$ when $PQ < 0$. In this case Ω is real and waves are stable. However, when $PQ > 0$, $\Omega^2 < 0$ then $K^2 < (2Q/P)|\Phi_0|^2$ and waves are modulationally unstable. The maximum growth rate is obtained for $K = \sqrt{|Q/P|\,|\Phi_0|}$ and is given by $\gamma_{max} = \mathrm{Im}(\Omega)_{max} = Q|\Phi_0|^2$. It is seen that instability sets in for perturbation wavelength $\lambda > \lambda_c$, where $\lambda_c = 2\pi/K_c$ and $K_c = \sqrt{|P/Q|\,|\Phi_0|}$.

Now we discuss the possible localized solitary wave solutions of (29). Since the wave packet can be stable or unstable in different conditions of θ, k, σ, β and α. P and Q can both be negative or they can have different signs. The latter condition implies two types of stationary solutions of NLSE. To obtain the profile in both cases, let us put

$$\phi = \rho(\zeta,\tau)e^{[\iota\sigma(\zeta,\tau)]} \tag{37}$$

where ρ and σ are two real variables. Substituting (37) into (29) and separating the real and imaginary parts and solve the resulting equation for ρ and σ. In case of modulationally unstable wave with P and Q having the same signs, we obtain the following envelope soliton solution

$$\phi(\xi,\tau) = \rho_m \mathrm{sech}\left(\sqrt{\frac{1}{2}\left|\frac{Q}{P}\right|}\rho_m\zeta\right) \tag{38}$$

where ρ_m is constant and represents the nonlinear maximum amplitude. On the other hand with P and Q having the opposite signs, we have modulationally stable wave and obtain

$$\rho(\xi,\tau) = \rho_1\left(1 - b^2\mathrm{sech}^2\left(\sqrt{\frac{\rho_1}{2}\left|\frac{Q}{P}\right|}b\zeta\right)\right)^{1/2} \tag{39}$$

where $1 \geq b^2 = \rho_1^2 - (\rho_m^2)\rho_1^2$, ρ_1 is a constant. Equation (39) represents an envelope hole sometimes called a dark soliton. Such solution corresponds to the accumulation of density in a region where wave intensity is very low. The parameter b determines the depth of the modulation. Further, when $b = 1$, we have

$$\rho(\xi,\tau) = \rho_1 \tanh\left[\sqrt{\frac{\rho_1}{2}\left|\frac{Q}{P}\right|}b\zeta\right] \tag{40}$$

which is known as envelope shock.

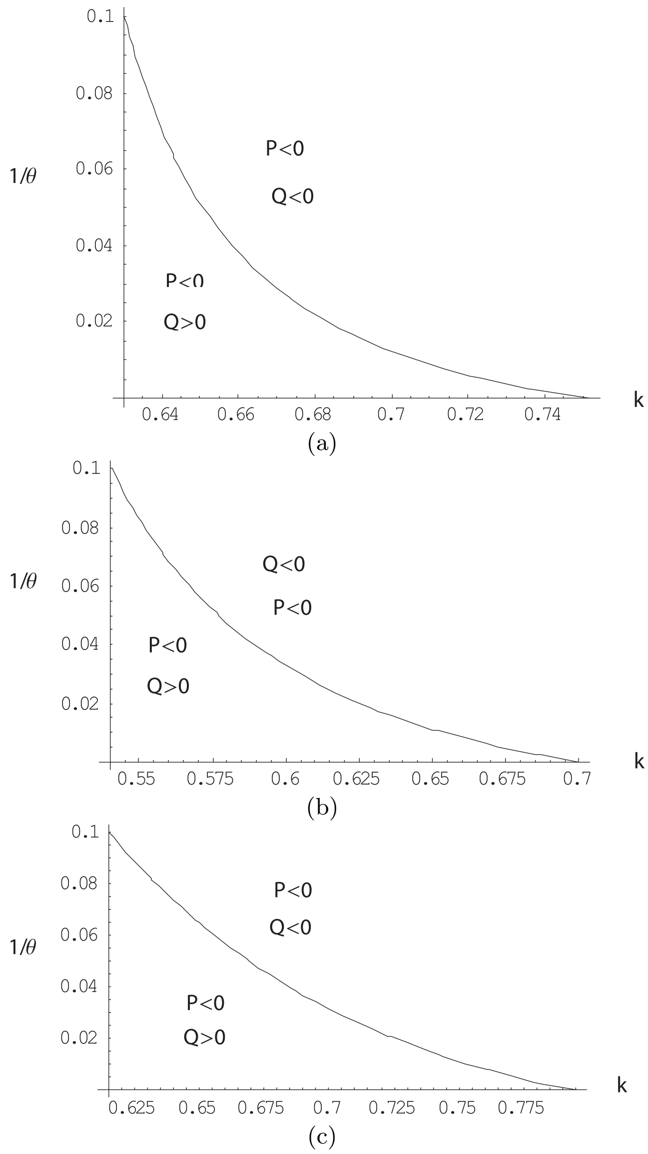

Fig. 2. Plot of $Q = 0$ in $(k - 1/\theta)$ plane for: (a) $\alpha = 2.0$, $\beta = 0$; (b) $\alpha = 2.0$, $\beta = 0.4$; (c) $\alpha = 2.0$, $\beta = 0.9$.

Coefficients P, Q of dispersion and nonlinear terms respectively are the functions of θ, nonthermal electrons distribution parameter β, ratio of hot electrons to cold electrons density α and wavenumber k. Therefore, one expects that θ and β will affect the unstable characteristics. We have chosen the following parameters:

$$n_{c0} = 0.5\ \mathrm{cm}^{-3}, \quad n_{h0} = 2.5\ \mathrm{cm}^{-3} \quad \text{and} \quad \beta = 0, 0.4, 0.9.$$

These parameters are within the range of observations from Viking satellite in the dayside auroral zone [10]. For pictorial presentation of the results, we perform computation and plot $Q = 0$ curves in $k - 1/\theta$ plane for different values of β as shown in Figure 2. Obviously, the parameter space $k - 1/\theta$ is divided into two regions i.e. $PQ > 0$ (unstable region) and $PQ < 0$ (stable region). Similar computation can be performed for the plot $P = 0$ curve in $k - 1/\theta$ plane for different values of β. It is noteworthy

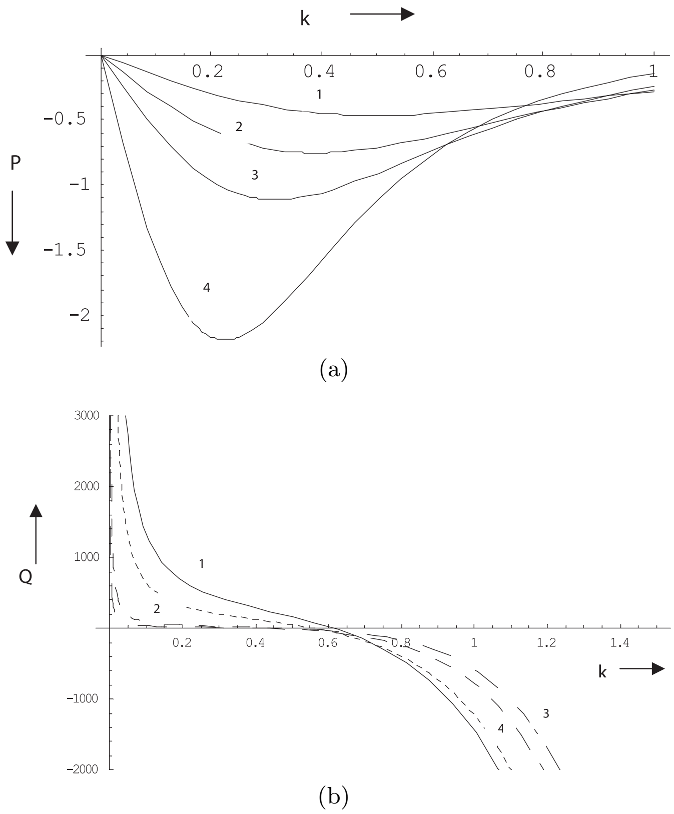

Fig. 3. Variation of P and Q as a function of k for four different values of β with $\alpha = 2.0$ and $\theta = 100$: for (a) curve 1 corresponds to $\beta = 0$, curve 2 to $\beta = 0.4$, curve 3 to $\beta = 0.6$ and curve 4 to $\beta = 0.8$; (b) curve 1 corresponds to $\beta = 0$, curve 2 to $\beta = 0.2$, curve 3 to $\beta = 0.6$ and curve 4 to $\beta = 0.8$.

that these two regions are significantly modified by nonthermal electrons distribution parameter β. Let us consider the case of isothermal electrons distribution corresponding to $\beta = 0$. In this case, we find that wave packet will be unstable at higher wave numbers ($k > 0.6$) and at higher T_c/T_h. The critical value of k for the onset of instability is lowered with increase of relative temperature T_c/T_h. This feature obviously highlights the crucial role of the nonthermal electrons distribution as a major contributing factor to cause the modulational instability. Figure 2 indicates the unstable region which is extended to lower wavenumber region with increase in value of β as long as $\beta < 0.4$. However, further increase of β reverses this trend as shown in Figure 2c.

Lastly, we have shown the variation of P and Q as a function of k for different values of β in Figures 3a and 3b respectively. From Figure 3a, it is obvious that width of the soliton increases with β. From Figures 3a and 3b, we find that the effect of β on wave amplitude and width is more significant for longer wavelengths.

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